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DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

THE DEVELOPMENT OF A THERMAL DISTORTION CODE USING THE FINITE ELEMENT METHOD

by

James H. Ma

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STRUCTURES DEPARTMENT
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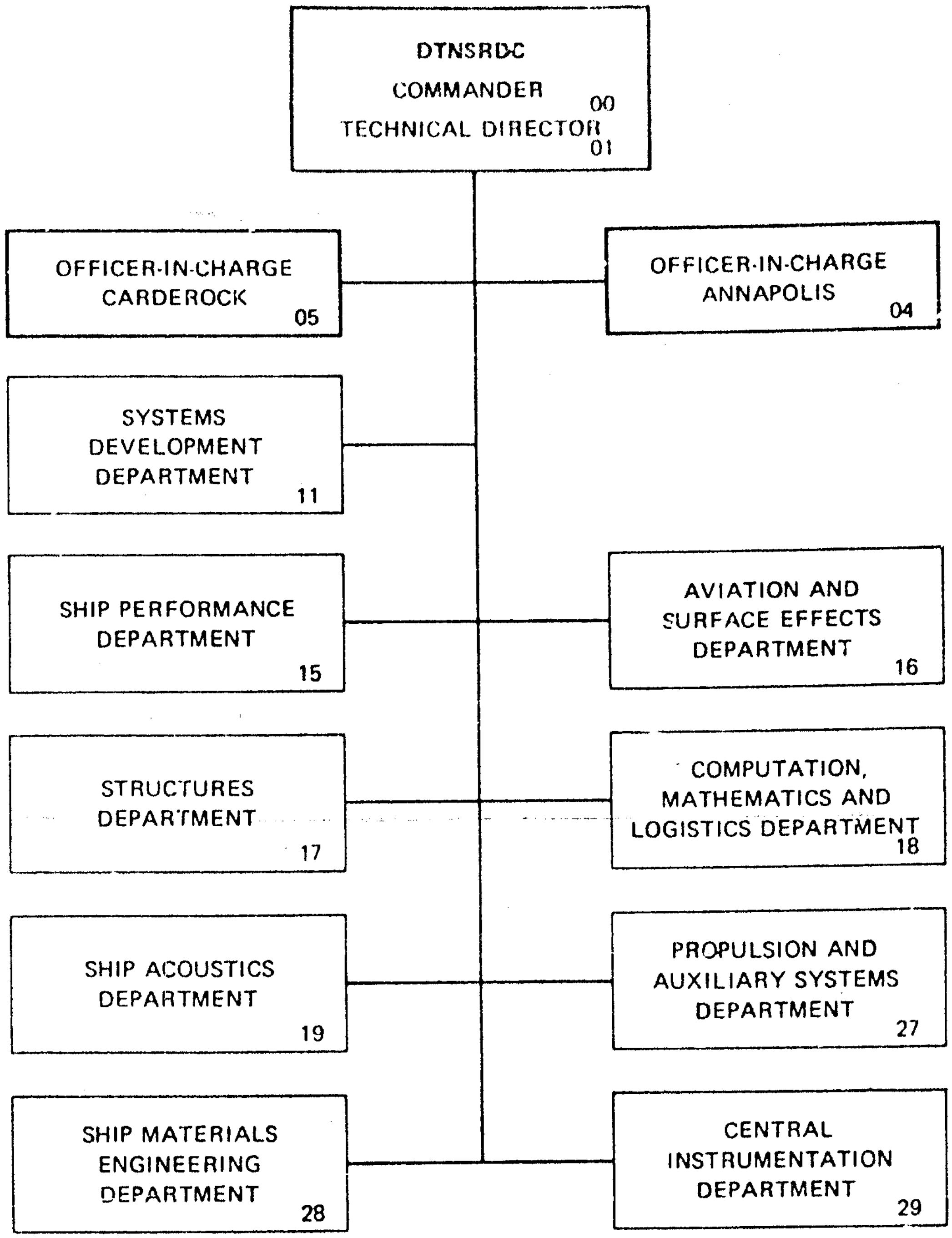

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A finite element code to account for thermal expansion in a solid was developed for the Independent Research and Independent Exploratory Development project "Tribology of Sliding Surface Bearings." The program is based on a two-dimensional model using a second, or higher order interpolation function in the element space that will allow a diverse

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temperature field, such as a steep, nonlinear temperature gradient, to be prescribed in a solid body.

The computer code has a definite advantage over certain finite element systems, commercially available. Many accept only a constant, or averaged temperature input into their element space. With the new capabilities, complex thermal mechanical responses under severe temperature gradients can be readily analyzed. For instance, the hot spot in a ship's landing deck due to the concentrated heat load, such as those generated by high temperature jet exhaust, can be more realistically represented by the elements of current development. The element mesh size and the input data set are more manageable.

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NOTATION

The symbols used in this study are defined where they first appear. For convenience, frequently used symbols are summarized below.

The bar and tilde generally denote a vector and a matrix, respectively, whether as underscores or overscores. Braces and brackets are used, respectively, to denote a vector and a matrix. For example, a column vector U^a can be written

$$\underline{U}^a = \{U^a\} = \begin{Bmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \cdot \\ \cdot \\ \cdot \\ \underline{U}_i \end{Bmatrix}$$

with its subvector

$$\underline{U}_i = \{U_i\} = \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix}$$

[B]	Matrix relating strain vector to nodal displacements
[D]	Elasticity matrix
E	Young's modulus of elasticity
{F}	A load vector
F_x, F_y, F_z	Element nodal force in the x, y, or z direction, respectively; forces are positive in the positive direction of x, y, or z axis
G	$= \dfrac{E}{2(1+\nu)}$ shear modulus of elasticity
H_{ix}, H_{iy}, H_{iz}	Weighting coefficient corresponding to position along Gaussian quadrature points ξ_{ix}, η_{iy}, or ζ_{iz}

I	Moment of inertia of a transverse section of a beam
i	Subscript indicating nodal number, or active index
$\hat{i},\hat{j},\hat{k}$	Vector having unit value in direction of x, y, or z axis, respectively
[J]	Jacobian matrix of coordinate transformation
$\|J\|$	Jacobian determinant
$[K_e]$ or $[K]^e$	Stiffness matrix of an element e
$N_i(\xi,\eta)$	Function of curvilinear coordinates, taking a value of unity at node i and zero at all other nodes
NNPE	Number of nodes per element
$\bar{p}$	Applied pressure on an element boundary
[SK]	Stiffness matrix of entire structure
TEMP(I),TE(I)	Temperature at node, I
TEMPF(J),TF(J)	Thermal expansion force at node I
TLOAD	Global thermal load vector (entire structure)
TREF	Ambient temperature, or arbitrary referenced temperature
t	Plate thickness
U	Vector of nodal displacement for entire structure
u, v	Component of displacement in the direction of the x or y axis, respectively, displacements are positive in the positive direction of coordinate axes
u_i, v_i	Component of displacement at node i
V, Vol	Volume of a given solid
X,Y,Z	Global system of rectangular coordinates
α	Coefficient of thermal expansion
γ_{xy}	Shearing strain components

ABSTRACT

A finite element code to account for thermal expansion in a solid was developed for the Independent Research and Independent Exploratory Development project "Tribology of Sliding Surface Bearings." The program is based on a two-dimensional model using a second, or higher order interpolation function in the element space that will allow a diverse temperature field, such as a steep, nonlinear temperature gradient, to be prescribed in a solid body.

The computer code has a definite advantage over certain finite element systems, commercially available. Many accept only a constant, or averaged temperature input into their element space. With the new capabilities, complex thermal mechanical responses under severe temperature gradients can be readily analyzed. For instance, the hot spot in a ship's landing deck due to the concentrated heat load, such as those generated by high temperature jet exhaust, can be more realistically represented by the elements of current development. The element mesh size and the input data set are more manageable.

ADMINISTRATIVE INFORMATION

This report covers work conducted for and funded by the David W. Taylor Naval Ship Research and Development Center's Independent Research and Independent Exploratory Development project, Tribology of Sliding Surface Bearings. The work was accomplished under Program Element 61152N, Task Area ZR 000 01 01, and Work Unit 2832-121.

BACKGROUND

PRELIMINARIES

The description of the behavior of solids under the action of heat and mechanical loads has been established in the field of thermoelasticity,[1*] however, the determination of the elastic or inelastic deformations in solid bodies under prescribed temperature distributions is only feasible for a number of elementary cases. It is difficult to generalize to more complex configurations commonly existing in real structures. Practical problems in the majority of cases are too complicated to allow an exact solution in the strict mathematical sense. Therefore, practicing engineers must either replace the actual structure in his analysis by a simpler one which can be solved or have recourse to an approximate method of analysis. In

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*A complete listing of references is given on page 21.

recent years, numerical methods, augmented by the powerful finite element technique, provide a realistic approach to many complex thermal mechanical problems.[2,3]

There is little doubt that good finite element codes exist, but users must be cautious that the performance of these codes is far from uniform. Generally, because finite element codes are not subject to regulations, there is no standard or warranty on their performance. In a recent report, a study was conducted on the thermal effects of an elementary type beam-column frame subjected to a temperature rise which varies linearly along the beam and virtually across sections normal to the beam. These thermal loads were superimposed with mechanical loads. A comparison was made of the results derived from the application of major finite element systems (using beam-type elements only) with those from hand calculations.[4] It is not surprising to discover that some well-advertised programs could not solve the problem and others handled the temperature effects in various ways which resulted in a large discrepancy in the computed answers.

All major finite element systems, commonly known as general purpose computer programs, possess certain thermal analysis capabilities. Most of them have demonstrated their heat transfer capabilities by providing solutions to radiation, convection, or heat conduction problems. In addition, problems involving thermal distortion have rarely received much properly documented attention.

A finite element code to account for thermal expansion in a solid was developed for the in-house research project "Tribology of Sliding Surface Bearings." The program is based on a two-dimensional model* using a second, or higher order interpolation function in the element space that will allow a diverse temperature field such as a linear or any nonlinear temperature gradient to be prescribed in a solid body.

This code has a definite advantage over certain commercially operated finite element systems which accept only constant or averaged temperature field. With the new capabilities, many practical problems can be readily solved, for example, the hot spot in the landing deck due to the concentrated heat load, such as those generated by high temperature jet exhaust, can be more realistically represented by the elements of current development. Element mesh size and input data set shall be more manageable.

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*A full 3-D finite element model is planned for the second phase of the current development.

SLIDING SURFACE BEARINGS

Figure 1 illustrates a crowned tilt pad bearing[5] situated under a hydrodynamic film, h, of lubricant. The purpose of the current study is the development of a computational program using the finite element technique to predict surface distortions for pivot-pad thrust bearings as a function of applied pressure and temperature gradient. Eventually the distortion analysis will form an integral part of a thermoelastic hydrodynamic computational scheme that will have the capability to predict the behavior of lubrication[6,7] even at its extreme conditions, such as the boundary lubrication.

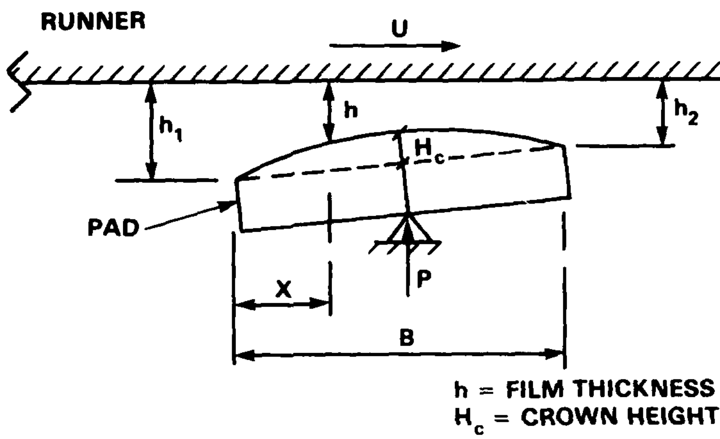


Figure 1 - A Crowned Slider Bearing Pad
(From figure by S. Abramovitz)

The performance of a hydrodynamic thrust bearing[8] or, more specifically, its load carrying capacity (i.e., the sum of the distributed film pressure over a bearing pad), depends to a large measure on the profile of the lubrication film which, in turn, is a function of the sliding velocity vector, the thermal coupling between the fluid film and the surrounding bodies, the thermal and elastic distortion or the crowning of the bearing surface, and, of course, the properties of the lubricant. The bearing sits tight on its pivot against its runner, which is a rotating collar and operates satisfactorily because it is sufficiently flexible to distort elastically and thermally.[6,9] The thermal distortions are of over-riding importance for these bearings.

These assertions outlined previously were substantiated by both theoretical and experimental evidence. It was known[10] that the load capacity of a truly flat centrally-pivoted pad operating in air is, theoretically, zero. The variation of viscosity (and density as well) of a liquid lubricant with temperature accounts for only a small part of the load capacity of such a pad operating in a liquid. Convexity, or the surface crown, inadvertantly obtained in machining and by load and temperature deflections in service, probably accounts for a large part of the observed load capacity of a centrally-pivoted pad. By controlling convexity, the load capacity of such bearings may be increased appreciably.

The standard procedure, used by lubrication engineers to analyze the distortions of the pad and thrust collar to give film shape, is solving the biharmonic equation of plate flexure;[9] and the finite difference approach[6,8] is normally followed. In this study, we set out a scheme that uses versatile continuum finite elements. It is envisaged that the new approach will remove all restrictions inherited from the plate theory, such as shear deformation in a thick plate, etc. Besides, the finite element solution will be advantageous in providing a precise representation of the complex pad geometry and temperature distribution, and the solution is numerically stable.

In a series of group conferences, the classical theory of thermoelastic hydrodynamic analysis[6,7] of the thrust bearing mechanism was evaluated, attention was focused on numerical techniques for the solution of finite bearings, and other alternative approaches were discussed. The major objective of the joint research program was to develop a reliable tool to predict the behavior of the thrust bearings in a wide range of operating conditions, i.e., in the hydrodynamic and boundary lubrications.

THERMOELASTIC FORMULATION

THEORY OF A FINITE ELEMENT SOLUTION

When a solid is under the environment of a thermal-mechanical loading, the total strains at each point of a heated body are made up of two parts. The first part is a uniform expansion proportional to the temperature rise, θ_e. Since this expansion is the same in all directions for an isotropic body, only normal strains, and no shearing strains, arise in this manner. If the coefficient of linear thermal expansion is denoted by α, this normal strain in any direction is equal to $\alpha\theta_e$. Hence, initial strains, due to thermal expansion in the case of a plane stress model, become

$$\begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \end{Bmatrix} = \begin{Bmatrix} \alpha\theta \\ \alpha\theta \\ 0 \end{Bmatrix} = \{\varepsilon_o\} \qquad \text{thermal strain vector} \tag{1}$$

The second part comprises the strain required to maintain the continuity of the body as well as those arising because of external loads. These strains are related to the stresses by means of the usual Hooke's law of linear elasticity. The total strains are the sum of these two components and are expressed[1,11] as follows:

$$\left.\begin{aligned} \varepsilon_x &= \frac{1}{E}[\sigma_x - \nu\sigma_y] + \alpha\theta \\ \varepsilon_y &= \frac{1}{E}[\sigma_y - \nu\sigma_x] + \alpha\theta \\ \gamma_{xy} &= \frac{1}{G}\sigma_{xy} \end{aligned}\right\} \tag{2}$$

where $G = E/2(1+\nu)$, and E and ν are elastic constants commonly known as Young's modulus and Poisson's ratio, respectively. For the range of temperatures encountered, e.g., bearing surface temperature, the material properties (α and E) are taken as constant (temperature independent) which appears to be adequate for current applications (see examples).

Following a typical finite element format,[3,12] derived from energy considerations, the basis of computation for an element is obtained in matrix notation.

$$\{F\}^e = \int_V [B]^T[D][B]\, dV\{\delta\}^e - \{F_\varepsilon\}^e - \{F_p\}^e \tag{3}$$

where $\{F\}^e$ is a vector of applied nodal forces

$\{F_\varepsilon\}^e = \int_V [B]^T[D]\{\varepsilon_o\}\, dV$ is the nodal force vector due to initial, imposed strain vector $\{\varepsilon_o\}$

$\{F_p\}^e = \int_V [N]\{p\}\, dV$ is the nodal force vector due to distributed loads $\{p\}$

$\int_V [B]^T[D][B]\, dV = [SK]^e$ is the element stiffness matrix

and

$\{\delta\}^e = [u_1,v_1,u_2,v_2,\ldots]^T$ is an element displacement vector

$$= \{\delta_i\}^e \text{ where } \delta_i = \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} i = 1, \ldots, \text{NNPE}$$

Then, NNPE is the number of nodes per element, for instance, NNPE = 8. The integration is normally carried out numerically as a summation of quadrature, such as Legendre-Gauss quadrature. A three-point integration along each coordinate line appears adequate.

Consider a general quadrilateral element having an arbitrary shape. Figure 2 depicts an 8-node element; its corner node numbers are 1 to 4 and the midside nodes

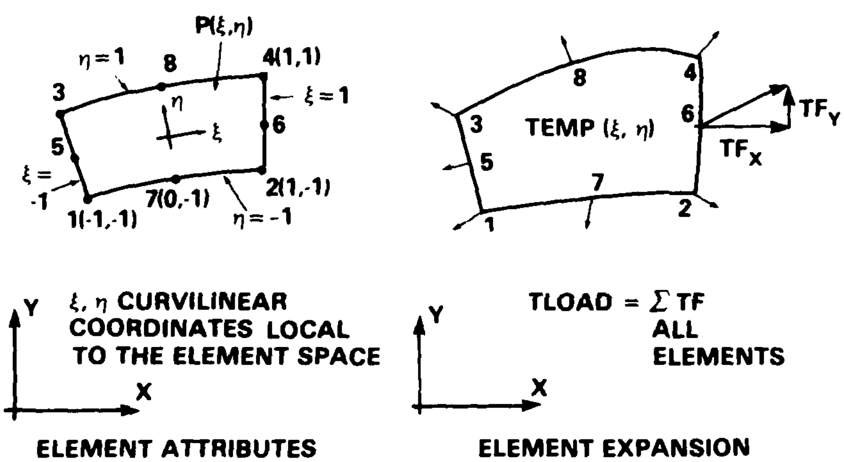


Figure 2 - A Curved Quadrilateral Element for Thermal Analysis

are 5 to 8. These nodes define the boundary of the element space. This element, commonly known as an isoparametric element,[3,12] is chosen for the current application for its proven performance and simplicity by which the temperature field, TEMP(x,y) in an element space, can be described.

$$\begin{aligned} \mathrm{TEMP}(x,y) &= N_1\ \mathrm{TE}_1 + N_2\ \mathrm{TE}_2 + \ldots + N_8\ \mathrm{TE}_8 \\ &= \mathrm{TEMP}(\xi,\eta) \end{aligned} \qquad (4)$$

where TE(I) is the temperature at node I, and $N_i = N_i(\xi,\eta)$ is the interpolation functions (or shape functions)[3] where ξ and η are curvilinear coordinates local to an element. In this example, a piecewise linear or a piecewise quadratic temperature field can be admitted.

The material components of a body will expand or contract as the temperature in the neighborhood of that material point rises or falls with respect to a reference temperature TREF, also referred to as the ambient temperature. That is,

$$\theta(x,y) = \mathrm{TEMP}(x,y) - \mathrm{TREF}$$

As the body of a continuum is idealized into finite elements and their connecting nodes, a set of nodal forces $\{F_\varepsilon\}$ representing traction or compression between the interface of adjoining elements will be generated from Equation (3).

$$\{F_\varepsilon\} = \begin{Bmatrix} \mathrm{TFX}_i \\ \mathrm{TFY}_i \end{Bmatrix} = t \int_{-1}^{1} \int_{-1}^{1} [B_i]^T [D] \{\varepsilon_o\}\ |J|\ d\xi d\eta \qquad (5)$$

$$i = 1,\ \mathrm{NNPE}$$

These nodal forces may, at times, be of a higher order than those surface loads which commonly occur. Special attention is called for to ensure that these force vectors are properly summed. Some detailed accounts on computations follow.

COMMENTS ON COMPUTATIONAL SCHEME

Consider the expansion load vector $\{F_\varepsilon\}^e$. In Equation (5), in which

$$|J| = \left|\frac{\partial(x,y)}{\partial(\xi,\eta)}\right| = \begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{vmatrix}$$

$|J|$ is the Jacobian determinant for coordinate transformation. To carry out this calculation, the quantities under the integral of Equation (5) must be defined at each integration point (ξ_{ix},η_{iy}). Begin with components of strain:

$$\varepsilon_x = \frac{\partial u}{\partial x} = \left[\frac{\partial N_1}{\partial x}, \frac{\partial N_2}{\partial x}, \ldots \frac{\partial N_8}{\partial x}\right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ \cdot \\ \cdot \\ \cdot \\ u_8 \end{Bmatrix} = \underset{1\times 8}{\frac{\partial \underline{N}_i^T}{\partial x}} \cdot \underset{8\times 1}{\underline{u}_i}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = \left[\frac{\partial N_1}{\partial y}, \frac{\partial N_2}{\partial y}, \ldots \frac{\partial N_8}{\partial y}\right] \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ \cdot \\ \cdot \\ \cdot \\ v_8 \end{Bmatrix} = \frac{\partial \underline{N}_i^T}{\partial y} \cdot \underline{v}_i$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial \underline{N}_i^T}{\partial y} \underline{u}_i + \frac{\partial \underline{N}_i^T}{\partial x} \underline{v}_i$$

Now, we can write the strain matrix [B] which relates the strain vector and nodal displacement vector of an element

$$\underset{3\times 1}{\{\varepsilon\}^e} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \underset{3\times 16}{[B]} \underset{16\times 1}{\begin{Bmatrix} \underline{u}_i \\ \underline{v}_i \end{Bmatrix}} \tag{6}$$

and, explicitly,

$$\underset{3\times 16}{[B]} = \begin{bmatrix} \frac{\partial N_1}{\partial x}, & \frac{\partial N_2}{\partial x}, & \frac{\partial N_3}{\partial x}, & \dots & \frac{\partial N_8}{\partial x}, & 0, & 0, & 0, & \dots & 0 \\ 0, & 0, & 0, & \dots & 0, & \frac{\partial N_1}{\partial y}, & \frac{\partial N_2}{\partial y}, & \frac{\partial N_3}{\partial y}, & \dots & \frac{\partial N_8}{\partial y} \\ \frac{\partial N_1}{\partial y}, & \frac{\partial N_2}{\partial y}, & \frac{\partial N_3}{\partial y}, & \dots & \frac{\partial N_8}{\partial y}, & \frac{\partial N_1}{\partial x}, & \frac{\partial N_2}{\partial x}, & \frac{\partial N_3}{\partial x}, & \dots & \frac{\partial N_8}{\partial x} \end{bmatrix}$$

The shape functions $N_i = N_i\ (\xi,\eta)$ are described in local coordinates to differentiate with respect to global coordinates, we follow the standard rule of partial differentiation.

$$\frac{\partial N_i}{\partial x} = \frac{\partial N_i}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial N_i}{\partial \eta}\frac{\partial \eta}{\partial x}$$

and

$$\frac{\partial N_i}{\partial y} = \frac{\partial N_i}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial N_i}{\partial \eta}\frac{\partial \eta}{\partial y}$$

We can proceed to form these products by using the Jacobian matrix and its inverse at each integration point

$$[J]^{-1} = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\ \frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y} \end{bmatrix}$$

While forming the Jacobian determinant,* note that

$$|J| = \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \eta}\frac{\partial x}{\partial \eta}$$

$$= \left(\underline{X}_i^T \frac{\partial \underline{N}_i}{\partial \xi}\right)\left(\frac{\partial \underline{N}_j^T}{\partial \eta}\,\underline{y}_j\right) - \left(\underline{X}_i^T \frac{\partial \underline{N}_i}{\partial \eta}\right)\left(\frac{\partial \underline{N}_j^T}{\partial \xi}\,\underline{y}_j\right)$$

$$= \underline{X}_i^T \left[\frac{\partial \underline{N}_i}{\partial \xi}\frac{\partial \underline{N}_j^T}{\partial \eta} - \frac{\partial \underline{N}_i}{\partial \eta}\frac{\partial \underline{N}_j^T}{\partial \xi}\right] \underline{y}_j$$

and that the matrix

$$\left[\frac{\partial \underline{N}_i}{\partial \xi}\frac{\partial \underline{N}_j^T}{\partial \eta} - \frac{\partial \underline{N}_i}{\partial \eta}\frac{\partial \underline{N}_j^T}{\partial \xi}\right] = [Q_{ij}] \qquad (7)$$

It is seen that $[Q_{ij}]$ is skew symmetric, i.e.,

$$Q(I,J) = -\,Q(J,I) \text{ and}$$

$$Q(I,I) = 0 \qquad \text{for } I,\ J = 1.8$$

In addition,

$$\left.\begin{aligned} \frac{\partial N_j}{\partial y} &= \frac{\underline{X}_i^T\,\underset{\sim}{Q}(ij)}{|J|} \quad \text{sum on } i \\ &= \frac{\underline{S2(j)}}{|J|} \end{aligned}\right\} \qquad (8)$$

(cont.)

*To have an inverse of the Jacobian matrix implies $|J| \neq 0$, because the Jacobian index is characterized on an element geometry.[12] The user of such an element should not allow the quadralateral to be in a degenerated form.

and

$$\left.\begin{aligned} \frac{\partial N_j}{\partial x} &= -\frac{\underset{\sim}{y}_i^T \, \underset{\sim}{Q}(ij)}{|J|} \qquad \text{sum on } i \\ &= \frac{S1(j)}{|J|} \end{aligned}\right\} \tag{8}$$

The Jacobian determinant is simply $\sum_{j=1}^{8} S2(j)\, y_j = |J|$.

Having the element strain matrix [B] defined term by term, we can summarize the expression for the thermal force vector as

$$\{F_\varepsilon\}^e = TF(16) = \begin{Bmatrix} TFX(8) \\ TFY(8) \end{Bmatrix}$$
$$= TEMPF\ (8,2)$$

From Equation (5),

$$TEMPF\ (8,2) = \int_{-1}^{1}\int_{-1}^{1} \frac{1}{|J|} \begin{bmatrix} S1(1) & 0 & S2(1) \\ S1(2) & 0 & S2(2) \\ S1(3) & 0 & S3(3) \\ \vdots & \vdots & \vdots \\ S1(8) & 0 & S2(8) \\ 0 & S2(1) & S1(1) \\ 0 & S2(2) & S1(2) \\ 0 & S2(3) & S1(3) \\ \vdots & \vdots & \vdots \\ 0 & S2(8) & S1(8) \end{bmatrix} \left(\frac{Et}{1-\nu^2}\right) \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \alpha\theta \\ \alpha\theta \\ 0 \end{Bmatrix} |J|\, d\xi d\eta$$

(9)
(cont.)

$$= \sum_{i\eta} \sum_{i\xi} \begin{bmatrix} S1(i) \\ \\ S2(i) \end{bmatrix}_{16\times 1} (1+\nu)\ \alpha\theta * DKT * H_{i\xi} * H_{i\eta} \qquad i = 1,8 \tag{9}$$

where $H_{i\xi}$, $H_{i\eta}$ is the weighting coefficient corresponding to position along Gaussian quadrature points ξ_{ix}, η_{iy}, respectively,

$$DKT = \frac{E * t * FACT\ (NEL)}{(1-\nu^2)}$$

t * FACT (NEL) is the thickness of the element number NEL

These thermal expansion forces, TEMPF, are the result of temperature rise, θ, inside an element space. They must be properly summed to form the thermal load vector, TLOAD, for the whole structure system, i.e.,

$$\left.\begin{aligned} TLOADX\ (NOD) &= \Sigma\ TFX\ (NI) \\ TLOADY\ (NOD) &= \Sigma\ TFY\ (NI) \end{aligned}\right\}\ \text{sum on elements}$$

where NI is a local node number attached to each individual element; for a given NI there is a corresponding global node number NOD. For each node, the summation shall extend to all elements that are connected to that node.

With these strain matrix-elements completely described at each integration point, the element stiffness matrix $[SK]^e$ can be immediately computed in much the same fashion. Finally, all those element matrices are assembled[12] for the complete structure. On imposing appropriate boundary conditions, a system of linear equations expressing the load displacement or equilibrium relation for the structure results. The system is then solved by the frontal solution technique.[12,13]

APPLICATIONS OF THE THERMAL DISTORTION CODE

Preliminary evaluations of the computed results on the element stiffness characteristics, as well as the nodal expansion force vector, for some basic element configurations and temperature profiles indicate that the present program has performed well in speed and reliability. In what follows, a few selected examples are given for illustration.

Thermal Bending of a Cantilever Beam

Thermal Distortion of a Tilt Bearing Pad

Structural Responses of a Ship Deck to Heat Load

THERMAL BENDING OF A CANTILEVER BEAM

A simple cantilever beam (Figure 3) was subjected to a linearly varying temperature gradient across the depth of the beam; no mechanical loads were applied. This example is taken from the ADINA[14] theoretical manual sample problem number seven in which the beam was modelled using three, 16-node solid elements. The analysis was carried out for material nonlinearities only, and, by using appropriate displacement boundary conditions, only the portion of the beam above the neutral surface was included in the finite element model. Partial results of ADINA's calculation are reproduced in Figure 4 showing the displacement response of the centilever neutral surface. Excellent agreement with the classical solution by Boley and Weiner[1] was obtained.

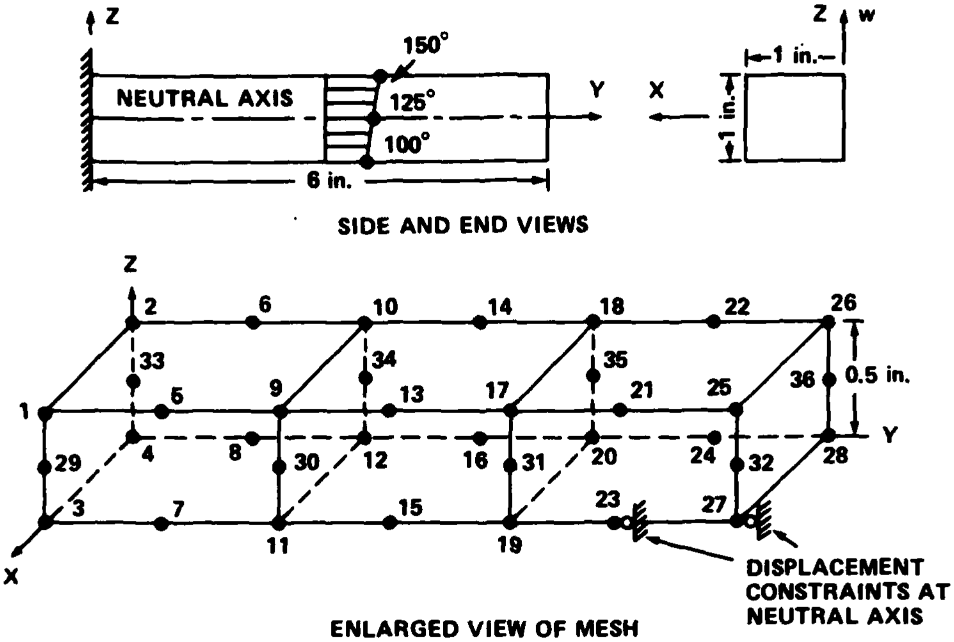
The ADINA's sample problem is used here to test the current development. For the present calculation, the complete beam is represented by three second-order quadrilateral elements. A constant thermal expansion coefficient, $\alpha = 0.000006$ in./in./°F, is used instead of a slightly lower coefficient, $\alpha_a = \alpha_a$ (TEMP), that was used in the ADINA example. As expected, the end deflection is slightly higher by the present calculation, $\delta_e = 0.0054$ in. (against 0.0052 in. by ADINA). The complete displacement response of the sample beam problem is superimposed on Figure 4 (deflections, δ is measured along -Z direction). The performance of the current development appears favorably in terms of speed and reliability.

THERMAL DISTORTION OF A TILT BEARING PAD

The heat energy was generated by the flowing lubrication oil squeezed between the sliding bearing surfaces. Part of the heat was absorbed by the solid bearings and the balance was carried away through convection of the lubrication fluid into the oil grooves. For the thrust bearing at given operating conditions of speed and load, the temperature distribution within a pad will converge to a pattern satisfying the equation of heat balance. To demonstrate the performance of the finite element code, a set of input temperatures, simulating a bearing pad under a test situation, is chosen as input data (Tables 8 and 9 from Reference 15).*

The dimensions and material properties of the bearing pad, together with a set of temperature data are shown in Figure 5. Four 8-node quadrilateral elements connected to a set of 23 nodal joints are employed for its representation. The

*The pad geometry follows the description of T. Daugherty's memo (Code 2832) dated 4 Dec 1980.



	70°F	200°F	
E	30×10^6	28×10^6	lb/in.^{2}
ν	0.0	0.0	
α	5.5×10^{-6}	6.0×10^{-6}	in./in./°F

REFERENCE TEMPERATURE = 125°F = T_o

Figure 3 - Finite Element Mesh of a Cantilever Beam

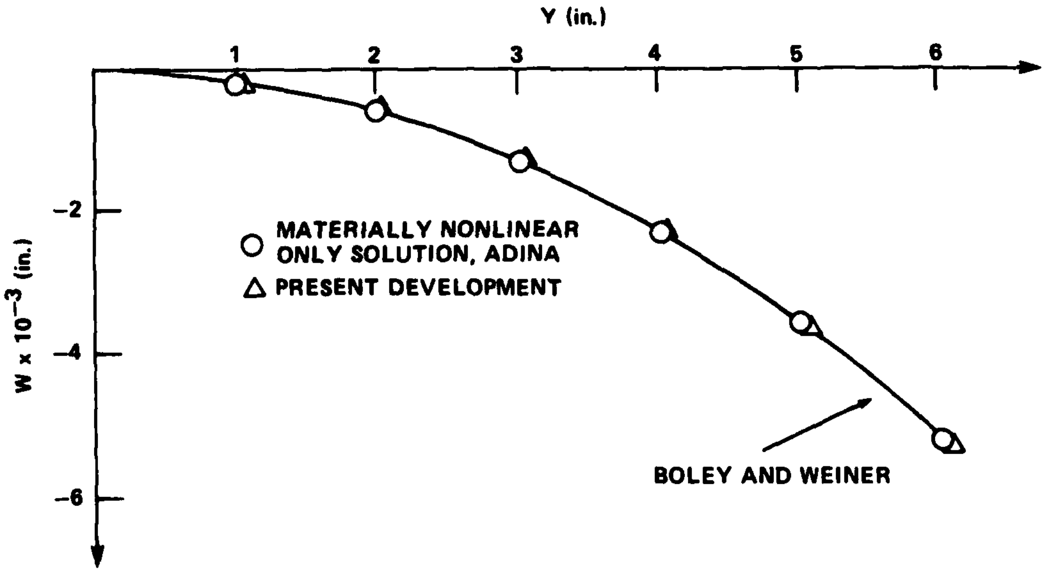
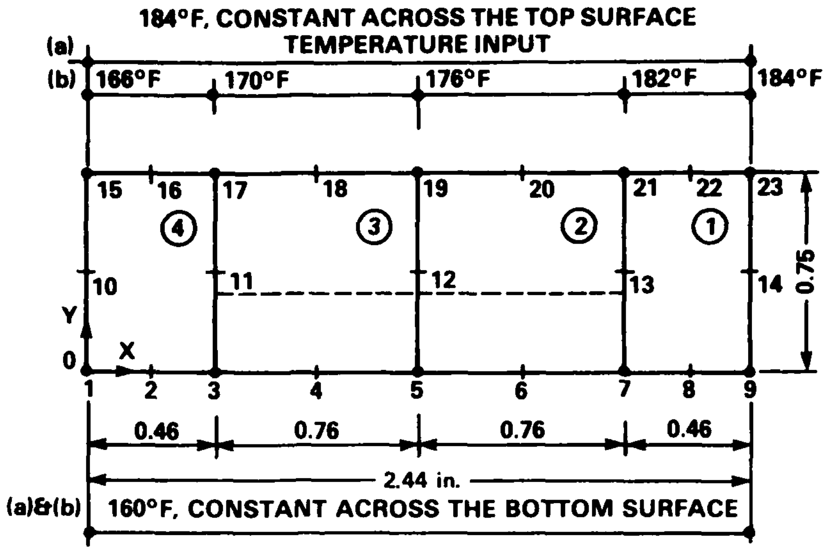


Figure 4 - Elastic Displacement Response of Cantilever



MATERIAL PROPERTY: E = 29,000 ksi ν = 0.30

TREF = 70°F $\alpha = 6.0 \times 10^{-6}$

Figure 5 - Bearing Pad Geometry and its Finite Element Representation
(With temperature input)

top surface is subjected first to a constant temperature of 184°F shown on line a, and then to a variable temperature increasing from 166°F to 184°F along the direction of flowing lubricant on line b. For the bottom face, a constant temperature of 160°F is maintained in both cases. The temperature gradient is assumed uniform across the thickness of the pad, otherwise temperature varies linearly between corner nodes in each element. The base line of the finite element model is supported against vertical movement, but the pad model is allowed to expand laterally along the OX-axis and vertically parallel to the OY-axis.

Selected numerical results of surface distortions, namely, displacement components Y, are plotted in Figure 6. Curves (a) predict the situation when the pad is subjected to a uniform temperature gradient across the pad width. Here the thermal distortion consists of a uniform thermal expansion and a simple thermal bending. The effect of thermal bending is a symmetrically crowned surface having a maximum offset at the midspan of 0.00060 - 0.00045 = 0.00015 in. which agrees very well with the hand calculation of 0.000155 in. based on the thermal bending formula.[9] Curves

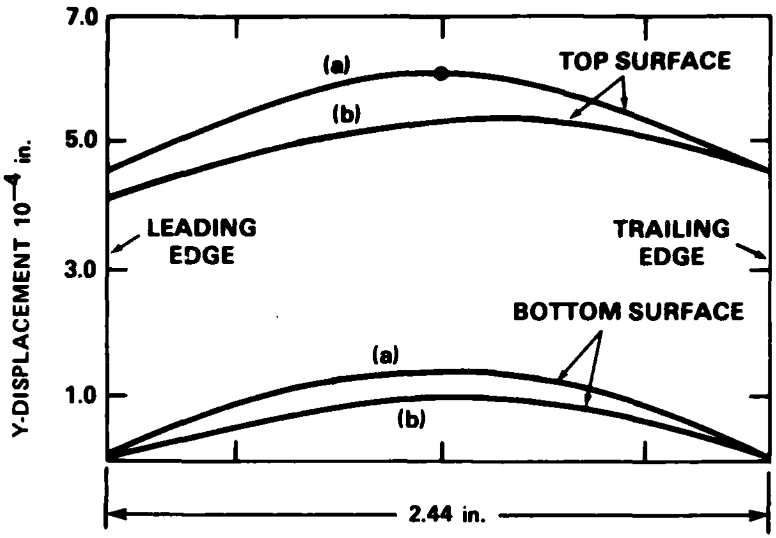


Figure 6 - Thermal Distortions of a Bearing Pad

(b) indicate the distorted surfaces of the pad under a skewed temperature load, line b in Figure 5. As a result, the surface crowning shifts towards the trailing edge. For the top surface, a crown height of 0.00010 in. occurs about 1.41 in. from the leading edge.

STRUCTURAL RESPONSES OF A SHIP DECK TO A HEAT LOAD*

A ship's deck can vary in size and shape, and differ in the method of construction and design arrangement. However, typically the deck consists of a welded steel** plate roughly 1/2- to 1-in. thick depending on its functional requirement. The plate is cross stiffened longitudinally and transversely by Tees ranging in size from 10 to 18 in.; these T-beams are, in turn, framed into stiffened wall panels. Further, a system of bulkheads, stanchions, and intermediate wall panels were installed to augment the load-carrying capacity of a deck structure. It is clear that the combination of these support systems results in deck construction which is

*This problem was funded by the surface ship structure block program, Task Area SF 43 400 391 and Work Unit 1730-035.

**Alternatively aluminum constructions were employed.

highly redundant. Presently, our interest is to identify the primary structural behavior of a deck subjected to a thermal loading. We shall focus our attention on a basic transverse beam-column structure unit.

Figure 7 shows a transverse bent composed of a 3/4-in. thick deck plate stiffened by an 18 in. by 0.5 in. web plate forming a Tee section girder having a moment of inertia, $I_g = 488$ in.^{4} The column has a cross section of 12 in. by 0.5 in. of which the moment of inertia, $I_c = 72$ in.^{4} The bent has an outside-to-outside width of 28 ft and head-room of 8 ft to the deck below.

For simplicity, we set up symmetric temperature data which somewhat resemble the heating load impinging upon a HELO-DECK due to the exhaust gas from a moderate-sized V/STOL (Vertical, Short Take Off and Landing Craft) such as the AV8 Harrier. Temperature could rise to a much higher level if the deck is exposed to the jet exhaust of a high performance major V/STOL craft.

A maximum surface temperature of 250°F occurs at midsection near the heating source. It drops off quickly to 130°F at a distance 35 in. to 50 in. away; then gradually reduces to 95°F near the edge of the deck. The under side of the deck plate has a maximum temperature of 170°F at the midspan and tapers off gradually to 90°F near the edge. Approximately linear temperature variation is assumed across the thickness of the deck plate. Other parts of the structural components will have an ambient temperature of 70°F.

For a symmetric heating, we need to consider only half of the transverse bent which can be represented by twelve 8-node elements* and a total of 56 nodal joints in Figure 8. Let us assume the midsection of the bent rests on top of a bulkhead which provides a solid support. If we leave the bottom of the bent free to move, it will have a distorted shape, as shown in Figure 9. At node 54, the center of the bottom edge, there is a movement of 0.4628 in. ($\Delta X^{54} = 0.3143$ in., $\Delta Y^{54} = 0.3391$ in.) and a clockwise rotation of 0.0031 radians. If we move the same point, node 54, back to its original position, a reactive force of 3.422 kip ($Rx = -0.106^k$, $Ry = 3.42^k$) is required to maintain the structural continuity, i.e., between the bent structure and its supporting deck (01 level).

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*A more elaborate finite element model composed of 120 elements or more can be set up when the loading situation calls for such an analysis.

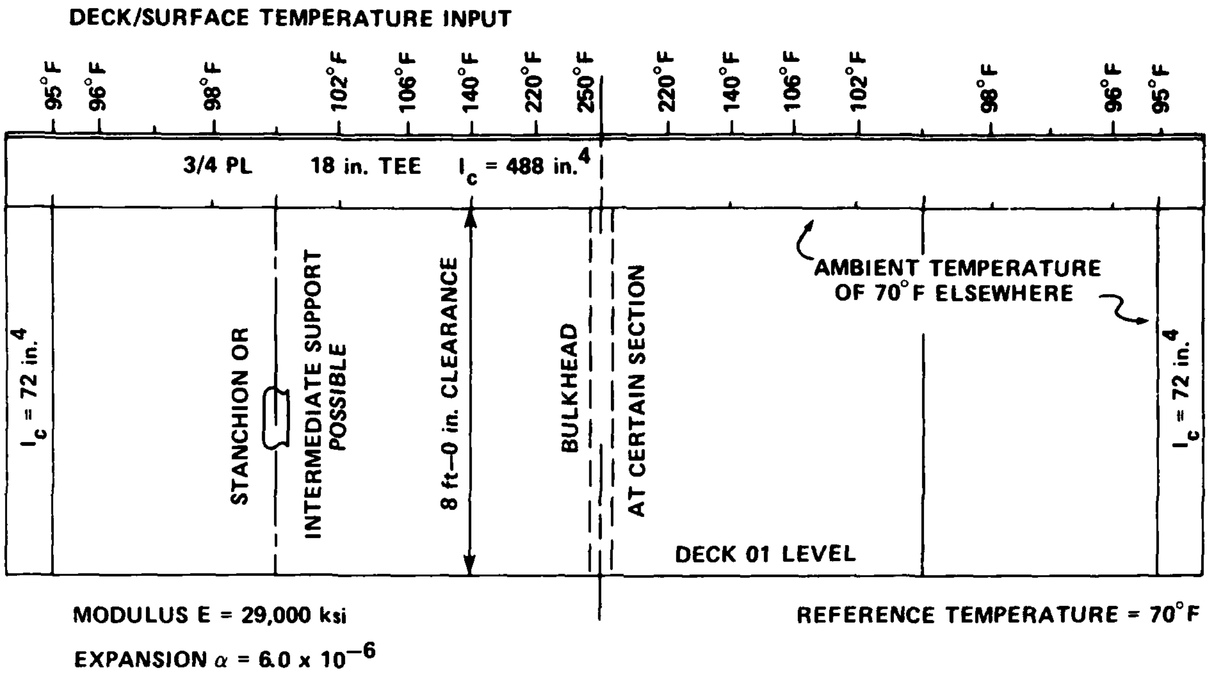

Figure 7 - A Transverse Bent of a Ship Deck

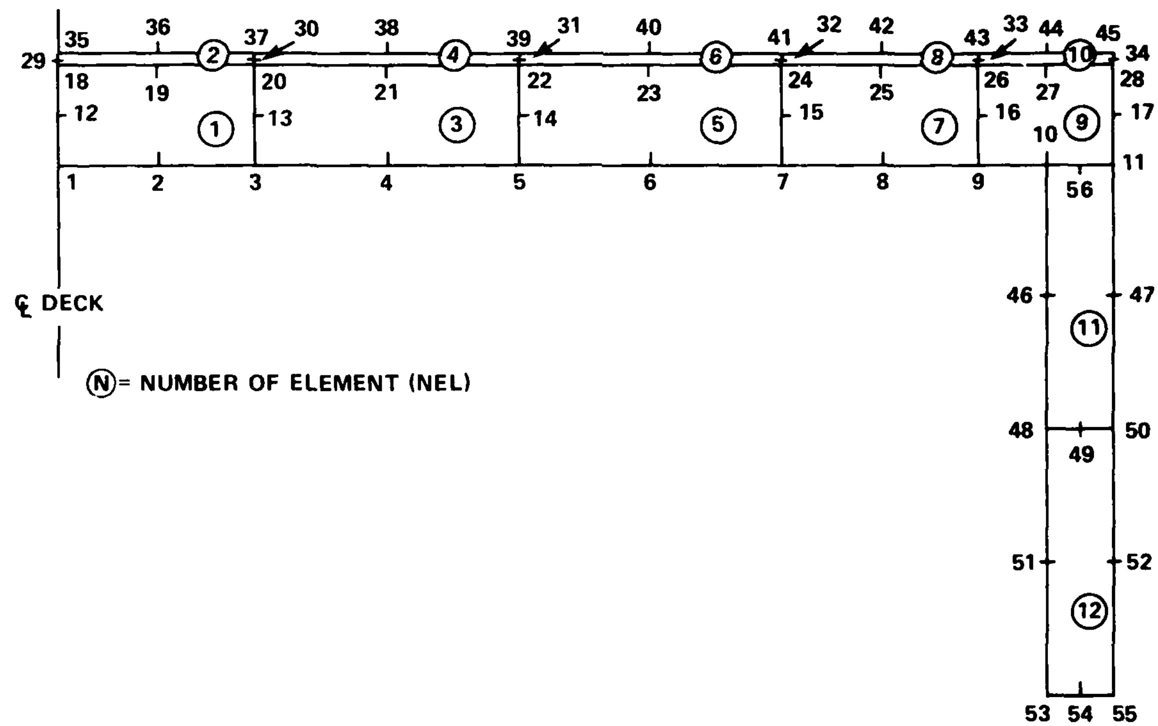

Figure 8 - A Finite Element Model of One-Half of a Bent Structure

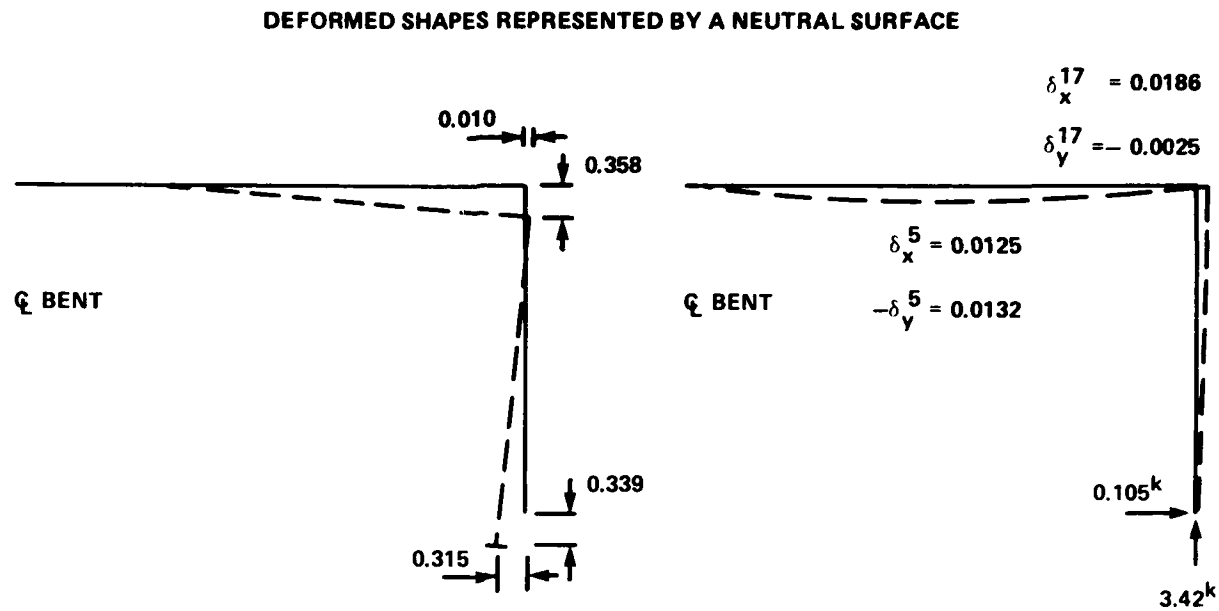

Figure 9a - Base is Free

Figure 9b - Base is Supported

Figure 9 - Distortions of a Half Transverse Bent by Deck Heating

CONCLUSIONS

A thermal distortion analytical capability has been implemented in a finite element computer code. The numerical results have correctly predicted the structural responses to a variety of thermal loadings. It is known that thermal distortions are of over-riding importance to the performance[6] of a tilting pad bearing. The present code capable of representing precisely both a linear and even an arbitrary temperature distribution, furnishes an imperative feature to the execution of a joint task on the thermoelastic-hydrodynamic calculation of sliding surface bearings.

Recent studies on thermal structural responses also indicate some inconsistencies in the thermal-structural capabilities of some major finite element systems[4] that have been available commercially. The present development, even though set up on a linear thermoelastic framework with temperature independent material properties, can be adopted to identify shortcomings that may exist in certain major finite element systems. In addition, the present code will provide an independent means capable of assessing rapidly a large number of structural responses under the prescribed thermal environments to which only a numerical solution offers a viable approach.

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